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BBA-003-001308 Seat No. ________

B. Sc. (Sem. III) (CBCS) Examination

July – 2021

Mathematics : Paper - BSMT - 301 (A) (Theory)

(Old Course) (Linear Algebra, Calculus & Theory of Equations)

Faculty Code : 003
Subject Code : 001308

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate full marks.

1 Answer the following questions in short : (1 Marks Each) **20**

(1) State whether the sentence is true or false : "A matrix has rank zero if and only if it is a zero matrix".

(2) Define : Subspace.

(3) "A vector containing a zero vector is always Linearly dependent" : True or False.

(4) $Dim(P_n) =$ _______ where P_n is set of all polynomials of degree $\leq n$.

(5) "If $\overline{0}$ is a zero vector and T is a transformation satisfying $T(\overline{0}) \neq \overline{0}$ then T is not a linear transformation" : True or False.

(6) State Rank-Nulity Theorem.

(7) Discuss the convergence : $1 = \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \ldots$

(8) "The series $\sum_{i}^{n} \frac{1}{n!}$ is divergent" : True or False.

(9) State : The conditions of convergence for D'Alembert's Ratio Test.

(10) Discuss the convergence : $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$

(11) Write the degree of the algebraic equation : $x^2 - 6x^3 + 8x - 4 = 0$

(12) A real root of the equation : $x^3 - 2x - 5 = 0$ lies in (a, b) then a = _______ and b = ________.

(13) Write the formula to find reciprocal of $\sqrt{N}$.

(14) Write the name of any one numerical method to find the derivatives of polynomial.

(15) The order of convergence in Newton-Raphson method is ________.

(16) Define : Radius of Curvature.

(17) What happens to formula of radius of curvature if the tangent at any point is parallel to y-axis ?

(18) Define : Double Point.

(19) Define : Cusp.

(20) The radius of curvature of the curve $y = e^x$ at the point where is crosses the y-axis is _______.

2 (a) Answer the following (any three) (Each Carries two marks) : **6**

(1) Check whether {(2, 2, 3), (2, 1, 3), (1, 0, 1)} is linearly dependent or not.

(2) If S is non-empty subset of vector space V then show that Sp S is subspace of V.

(3) Show that {(1, 1, 0), (1, 0, 1), (0, 1, 1)} is a basis of R^2.

(4) Find the Eigen values for $T : R^2 \to R^2$, $T(a, b) = (3b\ 2a - b)$.2y).

(5) Show that $T: R^3 \to \mathrm{R}^3$, $T\ (x, y, z) = (x^2, y^2, z^2)$ is a linear transformation.

(6) Write p-series and p-test

(b) Answer the following (any three) (Each Carries three marks) : **9**

(1) Check whether $(0, -1, 1) \in Sp\ A$ or not where $A = \{(2, 1, 0), (-1, 0, 1), (0, 1, 2)\}$.

(2) Let $S: U \to V$ and $T: U \to V$ be any two linear transformations, then show that $(S+T): U \to V, (S+T)(u) = S(u) + T(u)$, for $\forall_u \in U$ is also a linear transformation.

(3) Let $T: R^3 \to R^3$, be a linear transformation such that $T(1, 0, 0) = (0, 0, 1)$, $T(0, 1, 0) = (1, 0, 0)$ and $T(0, 0, 1) = (0, 1, 0)$ then prove that $T^2 = T^{-1}$ where $T^2 = T^\circ T$.

(4) Discuss the convergence : $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots$

(5) Examine the convergence : $\sum \sqrt{n^2+1}-\sqrt{n^2-1}$.

(6) Show that the series $\sum (-1)^n \left(\sqrt{n^2+1}-n\right)$ is conditionally convergent.

(c) Answer the following (any two) **5**
(Each carries five marks) :

(1) $\{V=(x, y)/x \in R, y>0\}$, for $(a, b),(c, d) \in V$ and $\alpha \in R:(a, b)+(c, d)=(a+c, bd), \alpha(a, b)=\left(\alpha\, a, b^a\right)$ then show that V is vector space.

(2) Show that $\{\bar{v}_1, \bar{v}_2, \bar{v}_3, \ldots, \bar{v}_n\}$ is linearly dependent iff $\exists\, \bar{v}_k \in V ; 2 \leq k \leq n$ such that $\bar{v}_k$ is linear combination of its preceding vectors $\bar{v}_1, \bar{v}_2, \bar{v}_3, \ldots, \bar{v}_{k-1}$.

(3) Let $T: R^3 \rightarrow R^3$ be a linear transformation such that $T(e_1)=e_1+e_2, T(e_2)=e_2+e_3$, where $\{e_1, e_2, e_3\}$ is standard basis of R^3 these find T^{-1}.

(4) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n \cdot x^n}{\sqrt{n+1}}$.

(5) Discuss the convergence : $\sum \frac{(n!)^2}{(2n)!} \cdot x^n$.

3 (a) Answer the following (any three) **6**
(Each Carries two marks) :

(1) Find the radius of curvature for the curve : $s=C \log (\sec \psi)$.

(2) Find the equation whose root is 1 more than the roots of the equation $x^3-5x^2+6x-3=0$.

(3) Find the radius of curvature for $xy - c^2$.

(4) Show that $y = \log x$ is convex upwards.

(5) Show that origin is a point of inflection for the curve $y = x^3, x \in R$.

(6) Find the asymptotes parallel to Y-axis for the curve : $x^2y^2 = a^2\left(x^2 + y^2\right)$

(b) Answer the following (any three) **9**
(Each Carries three marks)

(1) Derive Newton's Formula to find $\sqrt[p]{N}$.

(2) Derive Newton's Formula to find radius of curvature at origin for the curve $y = f(x)$.

(3) Find radius of curvature at any point (x, y).

(4) Find the asymptotes parallel to co-ordiante axes for the curve :

$x^2y - 3x^2 - 5xy + 6y + 2 = 0$

(5) Explain Newton-Raphson's method to find an approximate root of $f(x) = 0$.

(6) Show that the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ is $\frac{3a\sqrt{2}}{16}$.

(c) Answer the following (any two) **10**
(Each Carries five marks) :

(1) Obtain oblique asymptotes for the curve : $y = \frac{x^2 + 2x - 1}{x}$

(2) Find the radius of curvatures $p(r, \theta)$ for the curve : $r^2 = a^2 \cos^{2\theta}$.

(3) Explain False position method to find approximate root of $f(x) = 0$.

(4) Find the singular points for the curve :

$x^3 + y^3 - 12x - 27y + 70 = 0$.

(5) Find all the asymptotes for the curve :

$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 = 1$